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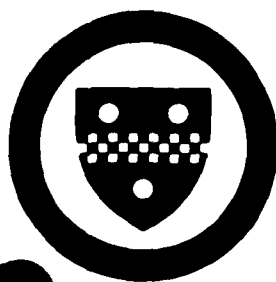
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Some Analytic Techniques for Parametrized
Nonlinear Equations and Their Discretizations¹⁾

by

James P. Fink and Werner C. Rheinboldt²⁾

1. Introduction

In many applications, nonlinear equilibrium problems typically involve a number of intrinsic parameters. Hence the resulting equations have the generic form

$$(1.1) \quad H(y, t) = y_0,$$

where y and t vary in some state space Y and parameter space T , respectively, and H is a given mapping with domain in $Y \times T$ and range in Y . In general, the set of all solutions (y, t) of (1.1) forms a manifold in $Y \times T$ and interest centers on analyzing the characteristic features of this equilibrium manifold.

Usually, the parameter space T has finite dimension, but the state space Y is infinite-dimensional. Thus for a computational analysis, finite-dimensional approximations of (1.1) have to be introduced, and then methods are required for determining the features of the solution manifold of the approximating finite-dimensional system of parametrized equations. At this time, the principal component of such methods always is a general form of continuation process for the trace of paths on the manifold. In addition, special procedures are available for detecting and determining specific features along these paths. The literature in this area is extensive, but there is no need to give specific citations since no use of them will be made. It should be noted, however, that there are only few studies about

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the integrated use of these various techniques for a thorough analysis of equilibrium manifolds.

The need for introducing finite-dimensional approximations of the equations (1.1) leads to further questions about the relation between the features of the solution manifolds of the original and the discretized equations and the errors introduced by the approximation. Relatively little has been done so far in this area, and, in fact, the series of articles [3], [5] appears to be the only one which addresses the a priori estimation of the errors in the case of a one-dimensional parameter space, and in [1] some a posteriori estimates of these errors are introduced for certain boundary-value problems in one space dimension.

In this paper, we outline some techniques for ensuring the existence of solution paths of a rather general class of nonlinear equations, as well as of their finite-dimensional approximations, and for assessing the error between these paths. A tool in this analysis is the theory of nonlinear Fredholm operators. The discussion of the operator discretizations involve a specific uniformity condition which represents a restriction of the class of operators used. The implication of this condition is the subject of ongoing research. But we show here that the results apply directly to mildly nonlinear elliptic boundary value problems of the type considered in [3], [5]. At the same time, it appears that they cover much more general classes of operators as well.

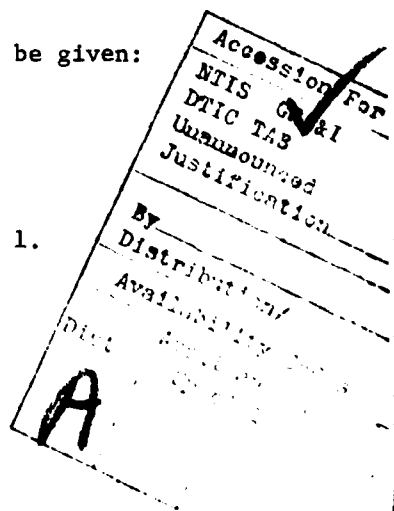
2. Solution Manifolds of Nonlinear Fredholm Operators

Throughout this paper the following information is assumed to be given:

- (i) two real Banach spaces X, Y ;
- (2.1) (ii) an open subset W of X ;
- (iii) a mapping $F: W \subset X \rightarrow Y$ of class $C^r(W)$, $r \geq 1$.

We are interested in examining the solution set of the equation

$$(2.2) \quad F(x) = y_0, \quad x \in W,$$



with a fixed $y_0 \in F(W)$. For this purpose, suppose that for some $m \geq 1$ the m -regularity set

$$(2.3) \quad R_m(F) = \{x \in W: \dim \ker DF(x) = m, \text{ rge } DF(x) = Y\}$$

of F is nonempty. Then the restriction of F to $R_m(F)$ is a nonlinear Fredholm operator of index m (see eg. [2]). Our analysis of (2.1) centers on this fact.

In [4] it was shown that in the case $m = 1$ the set $R_1(F)$ is open if it is not empty. A slight modification of the proof given there shows that if $R_m(F) \neq \emptyset$ for some $m \geq 1$ then again the set is open.

For any $x_0 \in R(F)$ there exist closed subspaces $V \subset X$ such that $X = V \oplus \ker DF(x_0)$. For any such choice of V the restriction $DF(x_0)|_V$ is an isomorphism between V and Y and the inverse

$$(2.4) \quad A_V = (DF(x_0)|_V)^{-1} \in L(Y, V)$$

is a right inverse of $DF(x_0)$ on Y . With this the equation (2.2) may be written in the form

$$(2.5) \quad F(t + A_V y) = y_0, \quad y \in Y, \quad t \in \ker DF(x_0), \quad t + A_V y \in W,$$

which corresponds to (1.1) with Y as the state space and $T_0 = \ker DF(x_0)$ as the m -dimensional parameter space.

The basic result about the solution set of (2.2) may now be phrased as follows:

Theorem 2.1: Suppose that (2.1) is given and that $R_m(F)$ is nonempty for some $m \geq 1$. Then for any fixed $y_0 \in F(W)$ the regular solution set $F^{(-1)}(y_0) \cap R_m(F)$ of (2.2) is a nonempty, open, m -dimensional C^r -manifold in X .

The proof is standard (see eg. a finite-dimensional analog given in [8], p. 11).

As mentioned in the introduction, the computational procedures for analyzing

the solution manifold of (2.2) consist principally of methods for approximating paths on that manifold. Such a path is defined as the solution manifold of some reduced equation defined by a Fredholm operator of index 1 on its regularity set. Thus without restriction of the generality we may assume that (2.1) is given and $R_1(F)$ is not empty. For ease of notation, we shall write $M(y_0) = F^{(-1)}(y_0) \cap R_1(F)$ for the open, one-dimensional C^r -manifold constituting the regular solution set of (2.2).

We consider first the question of the choice of suitable local parametrizations of $M(y_0)$. It turns out that a possible choice corresponds to a typical approach used in continuation procedures.

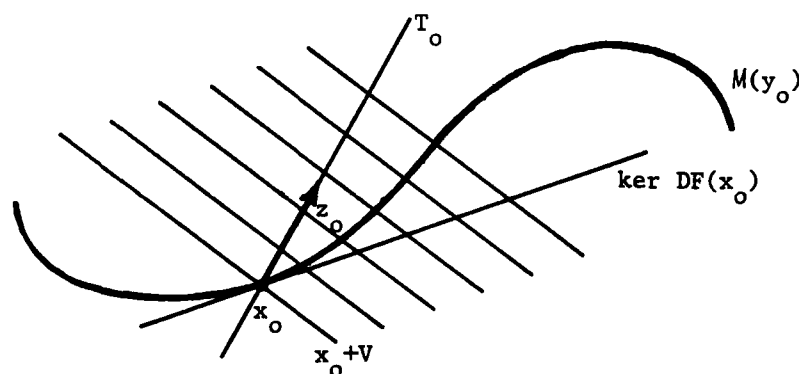


Figure 1

In line with the earlier discussion leading to (2.5), a local parametrization of $M(y_0)$ at a given point $x_0 \in M(y_0)$ is defined as a triple $\{V, A, z_0\}$ consisting of a closed subspace V of X , a linear map $A \in L(Y, V)$, and a point $z_0 \in X$ such that

- (2.6)
- (i) $\ker DF(x_0) \cap V = \{0\}$;
 - (ii) A is an isomorphism from Y onto V ;
 - (iii) $z_0 \notin V$;
 - (iv) $X = V \oplus T_0$, $T_0 = \text{span}\{z_0\}$.

As indicated in Figure 1, we consider the family of parallel linear manifolds

$$(2.7) \quad x_0 + tz_0 + V, \quad t \in \mathbb{R}^1.$$

The conditions (2.6) ensure that these manifolds are transversal to $\ker DF(x_0)$ and hence that locally near $t = 0$ the solution manifold may be parametrized in terms of t . This is the content of the following result:

Theorem 2.2: Suppose that (2.1) is given and $R_1(F) \neq \emptyset$. Moreover, let $\{V, A, z_0\}$ be a local parametrization of $M(y_0)$ at the point $x_0 \in M(y_0)$. Then there exists an open interval $J \subset \mathbb{R}^1$, $0 \in J$, an open neighborhood $U \subset X$ of x_0 , and a unique C^r -map $\eta: J \rightarrow Y$ such that

$$(2.8) \quad M(y_0) \cap U = \{x \in X: x = x_0 + tz_0 + A\eta(t), t \in J\}.$$

The proof follows from an application of the inverse function theorem and the use of the conditions (2.6).

In the standard continuation method, z_0 defines the predictor-line and the corrector produces the step $A\eta(t)$ from the predicted point $x_0 + tz_0$ to the point $x_0 + tz_0 + A\eta(t)$ on $M(y_0)$.

At any point $x_0 \in M(y_0)$ choose a nonzero vector $z_0 \in \ker DF(x_0)$. Then, as noted earlier, there exist closed subspaces $V \subset X$ such that $X = V \oplus \ker DF(x_0)$. With any such V and the associated mapping (2.4), the triple $\{V, A_V, z_0\}$ is a local parametrization of $M(y_0)$ at x_0 . In the setting of continuation methods, these parametrizations correspond, in essence, to the pseudo-arclength parametrizations (see eg. [6]).

The question now arises how far a local parametrization may be extended. For this, note that the set

$$(2.9) \quad A = \{x \in U: DF(x)A \text{ is an isomorphism of } Y \text{ onto itself}\}$$

is certainly nonempty and open. With this a generalization of a result in [7] may be phrased as follows:

Theorem 2.3: Under the conditions of Theorem 2.2, let $M_0 \subset X$ be the maximal con-

nected subset of $M(y_0) \cap A$ which contains x_0 . Then there exists an open interval $J_0 \subset \mathbb{R}^1$, $0 \in J_0$, and a C^r -map $\eta_0: J_0 \rightarrow Y$ such that

$$(2.10) \quad M_0 = \{x \in X: x = x_0 + tz_0 + A\eta_0(t), \quad t \in J_0\}.$$

The condition that, for $x \in M_0$, the map $DF(x)A$ is an isomorphism of Y means geometrically that our local parametrization is valid for the segment of the solution path between the points x_- and x_+ closest to x_0 , where $x_- + V$ and $x_+ + V$ are tangent to the path. In standard terminology these are the closest limit points of the path with respect to the direction z_0 .

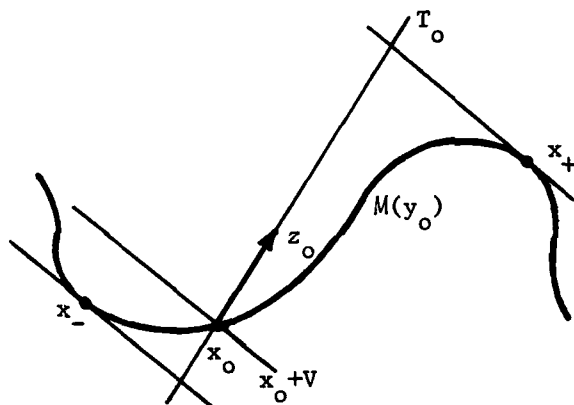


Figure 2

3. Finite-Dimensional Approximations

As before, suppose that the information (2.1) is given and that $R_1(F) \neq \emptyset$. We turn now to the formulation of suitable approximate problems for (2.2). In many applications, we have $X = Y \times \mathbb{R}$; that is, a particular component of X is identified as a basic parameter. Then only the complementary component Y has to be approximated. It is useful to generalize this by assuming that a splitting

$$(3.1) \quad X = Z \oplus T, \quad \dim T = 1$$

has been given together with a mapping

$$(3.2) \quad Q \in L(X, Y), \quad Q|Z: Z \rightarrow Y \quad \text{an isomorphism}$$

which relates Z with Y . Now a family $\{Z_h\}$ of finite-dimensional subspaces of Z , parametrized by some index $h > 0$, is assumed to be given and we set

$$(3.3) \quad X_h = Z_h \oplus T, \quad Y_h = QZ_h.$$

Moreover, let $P_h: Y \rightarrow Y_h$, $h > 0$, be a family of projections such that

$$(3.4) \quad \lim_{h \rightarrow 0} \|P_h y - y\| = 0, \quad \text{any } y \in Y.$$

With this our approximate problem is now specified by the equations

$$(3.5) \quad F_h(x) = y_{oh}, \quad x \in X_h, \quad y_{oh} = P_h y_o,$$

where

$$(3.6) \quad F_h: W_h \subset X_h \rightarrow Y_h, \quad F_h(x) = P_h F(x), \quad x \in W_h = W \cap X_h.$$

For ease of discussion, we call the information given in (3.1)-(3.6) a basic approximation of our problem (2.2).

As before, let $M(y_o) = F^{(-1)}(y_o) \cap R_1(F)$ be the solution manifold of the original problem. Our question is then whether the solution manifolds of the discretizations (3.5) approximate $M(y_o)$ when h tends to zero. For this analysis, we extend the discrete operators (3.6) to all of $W \subset X$ as follows:

$$(3.7) \quad \hat{F}_h: W \rightarrow Y, \quad \hat{F}_h(x) = (I - P_h)Qx + P_h(F(x) - y_o), \quad x \in W.$$

Clearly, \hat{F}_h is of class $C^r(W)$ for $h > 0$ and the following properties can be shown to hold:

- $$(3.8) \quad \begin{aligned} & (i) \quad \hat{F}_h(x) = 0 \quad \text{if and only if} \quad x \in X_h \quad \text{and} \quad F_h(x) = y_{oh}; \\ & (ii) \quad D\hat{F}_h(x)X_h \subset Y_h; \\ & (iii) \quad \ker D\hat{F}_h(x) \subset X_h; \\ & (iv) \quad D\hat{F}_h(x) \in L(X, Y) \quad \text{is a Fredholm operator of index 1;} \\ & (v) \quad P_h D\hat{F}_h(x)X_h = Y_h \quad \text{implies that} \quad x \in R_1(\hat{F}_h). \end{aligned}$$

Any comparison of the solution manifolds of (2.2) and (3.5) has to be done locally. Hence, let $x_0 \in M(y_0)$ be given and suppose that $\{V, A, z_0\}$ is a local parametrization of the manifold at x_0 . Clearly, this parametrization has to relate in a suitable way to the basic discretization introduced above. This relationship may be expressed in various forms. For the sake of brevity, we shall not go into details but assume here simply the technical condition

$$(3.9) \quad \|\hat{D}\hat{F}_h(x_0)Ay\| \geq \gamma \|y\|, \quad y \in Y, \quad h \in (0, h_0),$$

where $\gamma > 0$ is independent of y and h , and $h_0 > 0$ is sufficiently small. From (3.9) it follows that

$$(3.10) \quad P_h \hat{D}\hat{F}_h(x_0)X_h = Y_h$$

for $h \in (0, h_0)$ and hence, by (3.8)(v), that $x_0 \in R_1(\hat{F}_h)$.

The condition (3.9) may be enforced in many ways. For example, if we are prepared to restrict the discretization and parametrization by the choice $V = Z$, $A = A_V$, and $Q = A_V^{-1}Q_Z$, where $Q_Z \in L(X, Z)$ is the natural projection corresponding to the splitting (3.1), then $\hat{D}\hat{F}_h(x_0)A$ is the identity on Y and (3.9) holds with $\gamma = 1$ for any $h > 0$. On the other hand, as noted earlier, in many applications we have $Z = Y$, in which case we want to use $V = Z$, $A = I$, and $Q = Q_Z$, and (3.9) turns out to be a condition on F . One such case will be discussed in the next section.

Let $A_0: Y \rightarrow X$ be the affine mapping $A_0 y = x_0 + Ay$, $y \in Y$, and define $H_h: A_0^{(-1)}W \subset Y \rightarrow Y$ by $H_h(y) = \hat{F}_h(A_0 y)$, $y \in A_0^{(-1)}W$. Then, for $h \in (0, h_0)$, the condition (3.9) implies that $DH_h(0) \in L(Y)$ is an isomorphism and $\|DH_h(0)^{-1}\| \leq \gamma^{-1}$. By the uniform boundedness principle, the projections P_h are uniformly bounded for $h > 0$. Using this fact, we may show that there exists a $\delta > 0$, independent of h , such that

$$(3.11) \quad \|DH_h(y) - DH_h(0)\| \leq \frac{1}{2}\gamma \quad \text{whenever} \quad \|y\| < \delta.$$

This permits the application of the inverse function theorem. As noted in [3], the standard proof of that theorem permits the derivation of a Lipschitz estimate for the inverse function. Using a slight extension of the result in [3], we obtain here the existence of a unique C^r -function

$$(3.12) \quad G_h: U_h = B(w_{oh}, \frac{1}{2} \delta \gamma) \subset Y \rightarrow B(0, \delta) \subset Y, \quad w_{oh} = (I - P_h) Q x_o,$$

such that

$$(3.13) \quad H_h(G_h(y)) = y, \quad y \in U_h$$

and

$$(3.14) \quad \|G_h(y_1) - G_h(y_2)\| \leq \frac{2}{\gamma} \|y_1 - y_2\|, \quad y_1, y_2 \in U_h.$$

Since δ, γ are independent of h and $\|w_{oh}\| \rightarrow 0$ as $h \rightarrow 0$, it follows that there exists some $h_1 \in (0, h_o)$ such that $0 \in U_h$ for $h \in (0, h_1)$. Hence, $y_h = G_h(0)$ solves $H_h(0) = 0$, and from (3.14) we obtain that $\|y_h\| \leq (2/\gamma) \|w_{oh}\|$ for $h \in (0, h_1)$. Moreover, we can show that the corresponding points $x_{oh} + x_o + A y_h \in X$ satisfy $x_{oh} \in X_h$ and solve the approximate problem (3.5). Finally, it follows from (3.9) that $x_{oh} \in R_1(\hat{F}_h)$ and $\ker D\hat{F}_h(x_{oh}) \cap V_h = \{0\}$.

With this we have the following result:

Theorem 3.1: Let (2.1) be given and $R_1(F) \neq \emptyset$. Moreover, suppose that a basic approximation (3.1)–(3.6) has been chosen and that $\{V, A, z_o\}$ is a local parametrization of $M(y_o)$ at $x_o \in M(y_o)$ such that (3.9) is satisfied. Then, for all sufficiently small $h > 0$, the approximate problem (3.5) has a solution x_{oh} such that $x_{oh} \in R_1(F_h)$ and $\lim_{h \rightarrow 0} x_{oh} = x_o$. Furthermore, for any $z_{oh} \in X_h$, $z_{oh} \notin V_h = V \cap X_h$, it follows that $\{V_h, P_h DF(x_o)|_{V_h, z_{oh}}\}$ is a local parametrization of the solution manifold $F_h^{(-1)}(y_{oh}) \cap R_1(F_h)$ of (3.5) and hence Theorems 2.2 and 2.3 apply.

This result provides the existence of a local segment of the solution path of

the local approximate problem (3.5) under the same general conditions needed for establishing the existence of the solution curve of the full problem (2.2). However, if error estimates are desired, then additional smoothness conditions are needed for F . More specifically, the following extension of Theorem 3.1 can be proved:

Theorem 3.2: Suppose that the conditions of Theorem 3.1 hold and, in addition, that $DF: W \rightarrow L(X, Y)$ is Lipschitz continuous on bounded subsets of W . By Theorem 2.2, the original problem (2.2) has a solution segment $x: J \subset \mathbb{R}^1 \rightarrow W$ defined on an open interval $J \subset \mathbb{R}^1$, $0 \in J$. Then there exists a compact subinterval $J_0 \subset J$, $0 \in J_0$, such that, for sufficiently small $h > 0$, the local approximate problem (3.5) has a solution segment $x_h: J_0 \rightarrow W_h$ and

$$(3.15) \quad \|x(t) - x_h(t)\| \leq C \|(I - P_h)Qx(t)\|, \quad t \in J_0,$$

with a constant $C > 0$ which does not depend on $h > 0$ or $t \in J_0$.

The proof uses a particular representation of the two solution curves and is based on a globalized form of the implicit function theorem given in [3].

4. Mildly Nonlinear Operators

As an example, we apply our results to the mildly nonlinear operators considered in [3]. In line with the comments in the previous section about the condition (3.9), suppose that $X = Y \times \mathbb{R}$ and denote by $Q \in L(X, Y)$ the natural projection corresponding to this splitting. Let \hat{X} be another Banach space, $K \in L(\hat{X}, Y)$ a compact operator, and $G: X \rightarrow \hat{X}$ a given nonlinear mapping of class C^r , $r \geq 1$. With this we assume that our problem (2.2) involves the operator $F: X \rightarrow Y$ defined by $F(x) = Qx + KG(x)$, $x \in X$. With the notation $x = (y, \lambda)$, $y \in Y$, $\lambda \in \mathbb{R}$, for the vectors $x \in X$, the problem can then be written in the familiar form

$$(4.1) \quad y + KG(y, \lambda) = y_0.$$

As in Section 3, we introduce a basic approximation. Then it can be shown that

for any local parametrization $\{V, A, z_0\}$ of $M(y_0)$ at $x_0 \in M(y_0)$, the condition (3.9) holds for all sufficiently small $h > 0$. Hence, for mildly nonlinear operators, our basic assumptions are generally satisfied, and therefore Theorems 3.1 and 3.2 apply.

It may be interesting to see what type of local parametrizations correspond to the cases of nonsingular points and limit points considered in the first two parts of [3]. A point $x_0 = (y_0, \lambda_0) \in X$ is nonsingular if $D_y F(x_0) = I + K D_y G(x_0)$ is an isomorphism of Y onto itself. Hence, in this case, it follows that for any $x_0 \in R_1(F)$ we have $Y \cap \ker DF(x_0) = \{0\}$, whence for a local parametrization we may choose $V = Y$, A the identity on Y , and $z_0 = (0, 1)$, the λ -direction. This corresponds to the choice in the first part of [3]. Clearly, our results apply.

A point $x_0 = (y_0, \lambda_0) \in X$ is a limit point if $D_y F(x_0)$ has a one-dimensional null space in Y and $D_\lambda F(x_0) \notin \text{rge } D_y F(x_0)$. For $x_0 \in R_1(F)$, this is equivalent with $\ker DF(x_0) \subset Y$. Let $u \in Y$ be such that $D_y F(x_0)u = 0$, $\|u\| = 1$. Then $\ker DF(x_0)$ is spanned by $(u, 0) \in X$ and we have $Y = \text{rge } D_y F(x_0) \oplus \text{span}\{u\}$, and $D_y F(x_0)$ is an isomorphism of $Y_1 = \text{rge } D_y F(x_0)$ onto itself. It now follows that, for $x_0 \in R_1(F)$, we may introduce a local parametrization with $V = Y_1 \times \mathbb{R}$, $z_0 = (u, 0)$, and, for instance, the isomorphism $A \in L(Y, V)$ specified by $Ay = (y_1, t_1)$, where $y = y_1 + t_1 u$, $y_1 \in Y_1$, $t_1 \in \mathbb{R}$. This is exactly the approach in the second part of [3] and again our results apply.

The example discussed here is not the only one to which our results apply. More about this will be presented elsewhere.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This paper presents general techniques based on the theory of Fredholm operators for analyzing the solutions of parametrized nonlinear equations and their finite-dimensional approximations. In particular, it is shown how to obtain the existence of solution paths, both for a general class of nonlinear equations and for their discretizations, and to develop error estimates. Finally, it is also shown that the results include existing results for mildly nonlinear problems.		

